Extraction of GO and diffraction components from Physical Optics in terms of line integrations of equivalent edge currents by Modified Edge Representation

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Abstract: The surface to line integral reduction of PO currents by using Modified Edge Representation (MER) was empirically proposed (1989). For the observer without the Stationary Phase Point (SPP) on the scatterer, this reduction was reinforced mathematically (2001). Recently we showed, for the observer with SPP inside the scatterer surface, the MER line integration around the SPP gives GO terms(2004). In this talk, these are unified to conclude that MER line integration along the periphery and inner SPP, extracts the diffraction and GO components of PO surface integrals, respectively, irrespective of the observer position.

1. Introduction

Physical Optics (PO) [1] is one of the most widely used methods for estimating the scattered electromagnetic fields at high frequency from surface S. PO performs the scattered field calculations by two well-defined steps. Firstly, the total induced currents are approximated in the Geometrical Optics (GO) sense, based upon the locality principle. Secondly, the scattered fields, thus obtained when the currents are integrated over scatterer surface using the radiation integral. The scattered fields, thus obtained, are free of discontinuities and singularities on geometrical boundaries (Shadow - Reflection boundaries) and caustics as are usual the case with GO and Geometric Theory of Diffraction (GTD)[2]. On the other hand, the surface radiation integral becomes computationally heavy in the high frequency and for the large size of the scatterer and also, it hides the physical interpretation of the problem. Therefore, many works have been presented with the aim of reducing radiation integral to the line one, using asymptotic, exact or theoretic developments as shown in [3] - [6]. The reduction to line integral expressions also gives important and effective tools for the mechanism extraction of PO errors [7].

One of the complicated issues for surface to line integral reduction is "how to separate the diffraction and GO terms". In asymptotic discussion with the base on the method of stationary phase, the line integration around the periphery of the scatterer is regarded as the diffraction components [8][9]. On the other hand, if the line integration is derived exactly based upon the Field equivalence theorem, it should be equivalent not to the diffraction but to the scattered or GO + diffraction components [4][5]. The GO contribution from the central part of the surface, if any, would also be expressed as if it emanated from the edge. The former is more natural than the latter in that the equivalent edge currents express the diffraction which is inherently generated at the edge; diffraction is believed to be local phenomenon and it seems possible to define the equivalent edge currents in terms of local parameters only. However, since the relation between the diffraction and the PO radiation integrals varies with the observation regions or the location of the stationary phase point (SPP) vs the integration area, the comparison between the line integration for diffraction and the PO surface integration is not straightforward.

Authors have empirically proposed unique concepts of equivalent edge currents for PO diffraction components, named Modified Edge Representation (MER) [9] [10]. It was mathematically reinforced for the observer which has no stationary phase point (SPP) on the surface of the scatterer [8]. Recently, MER line integration was conducted not only along the periphery of the scatterer, but also along the infinitesimally small contour around the SPP inside the surface of scatterer (SPP_i) if any [11]. It was numerically found that this gives the GO components.

This article summarizes these works and concludes two general observations for surface to line integral reduction in terms of MER.

- 1) MER line integration along the periphery corresponds to diffraction components of PO irrespective of observer position.
- 2) PO surface integration is decomposed into GO and diffraction terms, in terms of MER line integration along the infinitesimally small contour around SPP and along the periphery of the scatterer, respectively.

2. Modified Edge Representation for surface-to-line integral reduction

The scattering field E^s radiated from the surface currents is written as:

$$\mathbf{E}^{\mathrm{s}} = \mathbf{j} \frac{\mathbf{k} \eta}{4\pi} \oint_{\alpha} \hat{\mathbf{r}}_{\mathrm{o}} \times \left(\hat{\mathbf{r}}_{\mathrm{o}} \times \overline{\mathbf{J}} \right) \frac{e^{-\mathbf{j}\mathbf{k}\mathbf{r}_{\mathrm{o}}}}{\mathbf{r}_{\mathrm{o}}} \mathrm{dS}$$
(1)

where \mathbf{r}_{o} is the distance from the integration point to the observer and $\hat{\mathbf{r}}_{o}$ is the unit vector toward the observer. The

time factor $e^{j\omega t}$ has been suppressed and only the $O(1/r_o)$ term is adopted assuming the observer is in the distance much larger than the wavelength. In PO, the surface electric current, \overline{J} is approximated by $\overline{J}=2\hat{n}\times H^i$ where \hat{n} is the unit vector normal to the surface. The scattering field thus obtained is denoted by E_{PO} hereafter. The relation and the definition of PO scattering fields and diffracted fields are checked first. The scattered field is simply defined as total field minus incident field everywhere. PO surface integration is the approximation for the scattered field in this definition. On the other hand, the definition of diffracted fields (E^{diff}) varies with the observation positions in association with that of GO fields. The source and the scatterer define three regions in the space. Regions I and III are reflection and shadow region, respectively. The stationary phase points fall on the scatterer surface (SPP_i). Region II is the space for which SPP is outside of the surface (SPP_o). The diffracted field is related with PO radiation integral as in Table 1, where E^R and E^i indicate the GO reflection and the direct incident wave associated with SPP_i, respectively.



Region	E ^{Diff} _{PO}
Region I	$E_{PO} - E^R$
Region II	E _{PO}
Region III	$E_{PO} + E^{i}$

Table 1. Definition of diffraction field in each region

MER is the unitary vector ($\hat{\tau}$) defined to satisfy the diffraction law, at the point of interest, for given directions of incidence and observation. So, $\hat{\tau}$ may be calculated from:

$$(\hat{\mathbf{r}}_{i} + \hat{\mathbf{r}}_{o}) \cdot \hat{\boldsymbol{\tau}} = 0 \tag{2}$$

 $\hat{\mathbf{n}} \cdot \hat{\mathbf{\tau}} = \mathbf{0}$ Vector $\hat{\mathbf{\tau}}$ depends on the source-observer directions as well as vector normal ($\hat{\mathbf{n}}$) to the surface (local dependence) Figure 2 illustrates the MER vector $\hat{\mathbf{\tau}}$ defined at each point on the edge as well as that inside the surface for the later extension. Figure 3 gives the example of MER and the visualized diffraction from the edge of the 15 λ x 15 λ square plate. The source is a Hertzian electric dipole with moment (1,0,0) located at (0,0,5 λ) and observer is at (-3 λ ,5 λ ,15 λ). The vector $\hat{\mathbf{\tau}}$ changes its direction and coincides with the real edge, only at four diffraction points. The significance of MER is its use in the reduction of PO surface radiation integral to the line one. The equivalent

electric and magnetic line currents along the actual edge \hat{t} are defined using the modified edge $\hat{\tau}$ as follows.

$$M_{I}^{MER} = \frac{(\hat{r}_{o} \times J_{o}) \times \hat{\tau}}{j(1 - (\hat{r}_{o} \times \hat{\tau})^{2})(\hat{r}_{i} + \hat{r}_{o}) \cdot (\hat{\tau} \times \hat{n})} \hat{t} \quad (3) \qquad J_{I}^{MER} = \frac{\{\hat{r}_{o} \times (\hat{r}_{o} \times J_{o})\} \times \hat{\tau}}{j(1 - (\hat{r}_{o} \times \hat{\tau})^{2})(\hat{r}_{i} + \hat{r}_{o}) \cdot (\hat{\tau} \times \hat{n})} \hat{t} \quad (4)$$

$$INSIDE \qquad SURFACE \qquad NEW USE \qquad VSE \qquad VSE$$

Figure 2 Definition of modified edge $\hat{\tau}$



Figure 3 MER for a square plate

It is noted that J_{I}^{MER} and M_{I}^{MER} , are singular at the SPPs on the edge if any; where $(\hat{r}_{i} + \hat{r}_{o}) \cdot (\hat{\tau} \times \hat{n}) = 0$ For region II, where there are no SPP_i, the Stokes theorem as well as the asymptotic approximation apply and it was mathematically proved that the PO surface radiation integral in (1) can be reduced to the MER^{Edge} line integrals as[8]:

$$MER^{Edge} \cong jk\eta(\overline{A} + \overline{B})$$
(5)

where:

$$\begin{bmatrix} \overline{A} \\ \overline{B} \end{bmatrix} = \frac{k}{4\pi} \oint_{\Gamma} \begin{bmatrix} \hat{r}_{o} \times \hat{r}_{o} \times J_{I}^{MER} \\ \hat{r}_{o} \times M_{I}^{MER} \end{bmatrix} \frac{e^{-jk(r_{i}+r_{o})}}{r_{o}} dl$$
(6)

The high accuracy of this integral reduction were demonstrated in [8] for flat and curved surfaces. Referring to the definition of diffraction in this region from Table 1, it comes that MER^{edge} corresponds to diffraction component of PO, denoted by MER^{diff} hereafter.

As for the regions I and III where SPPs exist on the scatterer surface, the MER currents are singular there as is illustrated in Figure 4 and the mathematical treatment based upon Stokes theorem in [8] is no longer valid. Furthermore, the entity of MER^{edge} is not clear. The recent work [11] extends the MER line integration along the periphery to that along the infinitesimally small contour around the SPP_i (MER^{SPP}) as is shown in Figure 5. It is noted that J_I^{MER} and M_I^{MER} , are singular at the specular reflection points for region I or at the intersection point of direct incidence for region III, denoted by SPP_i in the figures. The line contour Γ' is introduced as the boundary between S' and S_o. The radius of S' is ρ' and the limit $\rho' \rightarrow 0$ is considered.



Figure 4 Position of Stationary Phase Point (SPP)

Figure 5 Surface to line integral reduction for region I or III.

The work [11] numerically showed that the line integration Γ' around the SPP gives E^R in region I while $-E^i$ in region III. Figure 6 shows the scattering geometry for a sphere where the contour for the line integration around the SPP at O with the radius ρ' is defined. Figure 7 shows in region I that as the contour becomes smaller ($\frac{1}{\rho'}$ becomes larger), MER^{SPP} is approaching to the value of GO reflection, which varies with the curvature ρ_s of the scatterer surface. We have confirmed also that in region III, MER^{SPP} converges to the direct incidence with opposite sign which precisely creates the geometrical shadow in the total field, irrespective of the surface curvature ρ_s .



3. Extraction of GO and diffraction from PO surface integration in terms of MER line integrations. -Discussions -

In this section, the results in [8] for region II and those in [11] for region I and III are combined and their significance is highlighted. In region II, it was mathematically proved that the PO surface integration E_{PO} is reduced to MER line integration along the periphery (denoted by E_{MER}^{Per}) [8]. Since $E_{PO} = E^{diff}$ in region II from Table 1, we can conclude

$$E_{MER}^{Per} = E^{diff}$$
(Region II) (7)

Then the surface to line integral reduction is discussed for regions I and III, with reference to Figure 5. To utilize the result in [8], the surface S will be divided into S' around the SPP_i, and the complementary one S_o . Now the surface to line integral reduction for S_o is considered. Since the PO currents are regular and finite everywhere, E_{PO} for S' vanishes and E_{PO} for S equals to E_{PO} for S_o . The result [8] is applied for S_o and we get

$$E_{PO} = E_{MER}^{Peri} + E_{MER}^{SPP}$$
(8)

The numerical results in [11] can be summarized as,

$$-E^{R} \quad (region I) \qquad -E^{R} \quad (region II) \qquad -E^{R} \quad (region II) \qquad -E^{R} \quad (region II) \qquad -E^{R} \quad (region III) \qquad -E^{R} \quad (regin I) \qquad -E^{R} \quad (regin I) \qquad -E^{R} \quad (regin I$$

Then we get the unified expression from (7), (9) and (10) with reference to Table 1 as

$$E_{MER}^{Per} = E_{PO}^{diff} \qquad (every \ region) \tag{11}$$

$$E_{MER}^{SPP} = E_{PO}^{GO} \qquad (if any) \tag{12}$$

We conclude that PO surface integration is decomposed into two MER line integrations, where E_{MER}^{SPP} around the SPP_i and E_{MER}^{Per} along the periphery correspond to GO and diffraction, respectively. Furthermore the substance of E_{MER}^{Per} for regions I and III has been identified to be diffraction component for the first time; more simply E_{MER}^{Per} corresponds to PO diffraction everywhere.

4. Conclusions

In this article, the surface to line integrals reduction of PO is discussed. PO is decomposed into two MER line integrations corresponding to the GO and the diffraction. At the same time, MER line integration along the scatterer periphery corresponds to diffraction component in every region of observation.

5. References

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